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Transverse parton distribution functions at next-to-next-to-leading order: the quark-to-quark case

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We present a calculation of the perturbative quark-to-quark transverse parton distribution function at next-to-next-to-leading order based on a gauge invariant operator definition. We demonstrate for the first time that such a definition works beyond the first non-trivial order. We extract from our calculation the coefficient functions relevant for a next-to-next-to-next-to-leading logarithmic Q_T resummation in a large class of processes at hadron colliders.

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Parton distribution functions (PDFs) describe the momentum distribution of quarks and gluons inside hadrons, and are essential inputs for physics program at hadron colliders such as the CERN Large Hadron Collider (LHC). Their usefulness resides in the factorization theorems [1] which separate physics at different length scales. While PDFs may depend on the 4-momentum of the parton (the so-called fully-unintegrated PDFs), most physical observables are only sensitive to the forward component of the parton momentum. To be more precise, it is convenient to introduce two light-like vectors n and $\bar{n}$ satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$, where n is along the beam direction of the hadron. Any 4-vector v^μ can then be decomposed as $v^\mu = \bar{n} \cdot v\, n^\mu/2 + n \cdot v\, \bar{n}^\mu/2 + v_\perp^\mu$. For an energetic parton along the n-direction, its momentum q^μ has the hierarchical structure $q = (\bar{n} \cdot q, n \cdot q, q_\perp) \sim Q\,(1, \lambda^2, \lambda)$, with $\lambda \ll 1$. For hard interactions, the smaller components $n \cdot q$ and $q_\perp$ can often be neglected. The resulting PDFs are referred to as "collinear PDFs", which only depend on the fraction $z = \bar{n} \cdot q/\bar{n} \cdot p$ with p being the hadron momentum.

The collinear PDFs can be defined rigorously in quantum chromodynamics (QCD) as matrix elements of certain non-local operators [2]. For example, the quark PDF is given by

$$\phi_{q/N}(z, \mu) = \int \frac{dt}{2\pi}\, e^{-izt\bar{n}\cdot p} \times \langle N(p)|[\bar{\psi} W](t\bar{n})\, \frac{\bar{\not{n}}}{2}\, [W^\dagger \psi](0)|N(p)\rangle \,, \quad (1)$$

where ψ is the quark field and W is a light-like Wilson line which renders the operator gauge-invariant. The PDFs are non-perturbative objects, but their renormalization group (RG) evolution can be determined perturbatively as long as the factorization scale $\mu \gg \Lambda_{\mathrm{QCD}}$. Their anomalous dimension functions (splitting functions) have been computed up to 3 loops [3].

Certain observables, such as the transverse momentum distributions in the production of the Higgs boson or gauge bosons, however, are sensitive to the perpendicular component $q_\perp$ of the parton momentum. One therefore needs to keep $q_\perp$ in the definition of the relevant PDFs, which are referred to as transverse-momentum-dependent PDFs (TMDPDFs) or simply transverse PDFs. Closely related to the transverse PDFs is the Collins-Soper-Sterman (CSS) formalism of transverse momentum resummation [4], which addresses the divergent behavior of fixed-order calculations at small transverse momentum. The CSS formula for the Drell-Yan process can be written in the form

$$\begin{aligned}\frac{d\sigma}{dQ^2 dQ_T^2 dy} &= \frac{4\pi^2\alpha^2}{9Q^2 s} \int \frac{d^2x_\perp}{(2\pi)^2}\, e^{-iQ_\perp\cdot x_\perp} \sum_q e_q^2 \\ &\times \exp\left\{ -\int_{\mu_b}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\frac{Q^2}{\bar{\mu}^2}\, A\big(\alpha_s(\bar{\mu})\big) + B\big(\alpha_s(\bar{\mu})\big) \right] \right\} \\ &\times \left[\overline{\mathcal{P}}_{q/N_1}(z_1, x_T)\, \overline{\mathcal{P}}_{\bar{q}/N_2}(z_2, x_T) + (q \leftrightarrow \bar{q}) \right] , \end{aligned} \quad (2)$$

where Q^μ is the momentum of the Drell-Yan pair, $x_\perp$ is the variable conjugate to $Q_\perp$ (also referred to as the "impact parameter" b in the literature), and $\mu_b = b_0/x_T$ with $b_0 = 2e^{-\gamma_E}$. The functions $\overline{\mathcal{P}}$ can be interpreted as transverse PDFs. For $x_T^2 \equiv -x_\perp^2 \ll 1/\Lambda_{\mathrm{QCD}}^2$, one can match $\overline{\mathcal{P}}$ onto the collinear PDFs

$$\overline{\mathcal{P}}_{q/N}(z, x_T) = \sum_i \int_z^1 \frac{d\xi}{\xi}\, C_{qi}\big(\xi, \alpha_s(\mu_b)\big)\, \phi_{i/N}(z/\xi, \mu_b)$$

with perturbatively calculable coefficient functions C_{qi}. These coefficient functions were determined to the next-to-leading order (NLO) in [4, 5]. The general structure of C_{ij} at NLO was obtained in [6] including C_{gi} relevant for Higgs production. Previous results beyond NLO were based on a modification of the CSS formula [7], introducing functions $\mathcal{H}_{ij\leftarrow ab}$, which are related to the convolutions of C_{ia} and C_{jb} functions. The next-to-next-to-leading order (NNLO) corrections to $\mathcal{H}$ were computed [8] for the Drell-Yan process and for Higgs production. The method used in these results is to assume the formula (2), and compare its fixed-order expansion with explicit calculations of the Q_T spectrum. It is however desirable to have an operator definition of the transverse PDFs, and compute the matching coefficient functions from the definition.

One may extend the definition (1) to the case of transverse PDFs as [9]

$$\mathcal{B}_{q/N}(z,x_T^2,\mu) = \int \frac{dt}{2\pi}\, e^{-izt\bar{n}\cdot p} \times \langle N(p)|[\bar{\psi}W](t\bar{n}+x_\perp)\,\frac{\bar{\not{n}}}{2}\,[W^\dagger\psi](0)|N(p)\rangle \,. \quad (3)$$

This definition, however, is problematic since new divergences associated with the light-cone propagators arise, which are not regulated in dimensional regularization. Therefore, one needs to supplement Eq. (3) with some extra regulator to make it well-defined. Generically, the result will then depend on this extra regulator, while in physical observables such dependence will cancel.

Different kinds of regulators have been proposed in the literature. The original paper [2] employed a non-light-like axial gauge. That was employed in subsequent NLO calculations [10]. More recently, Collins introduced a gauge-invariant definition utilizing non-light-like Wilson lines [11]. An equivalent [12] definition has been put forward in [13]. Variants of Smirnov's analytic regulator [14] were used in [9, 15, 16]. All these approaches were argued to be valid to all orders in perturbation theory based on factorization properties. However, explicit calculations were only carried out at NLO. In this Letter, we report the first NNLO result for the transverse PDF in the quark-to-quark case, using the regulator proposed in [15]. We also extract the NNLO coefficient function $C_{q\bar{q}}^{(2)}$, which is the first direct calculation of this function.

We consider processes where a $q\bar{q}$ pair annihilates into some color neutral final state F: $q(p_1)+\bar{q}(p_2)\to F(Q)$, with p_1 along the n-direction and p_2 along the $\bar{n}$-direction. We follow closely the formalism in [9], where the transverse PDF for quarks along the n-direction is defined as in Eq. (3), while the PDF for quarks along the opposite direction $\bar{\mathcal{B}}_{\bar{q}/N}$ is defined with $n\leftrightarrow\bar{n}$. To compute the matching functions, we replace the hadron field N with a quark field and evaluate the matrix element in Eq. (3). The $\mathcal{B}$ functions require an extra regulator beyond dimensional regularization, for which we adopt the one introduced in [15], namely, we multiply a factor $(\nu/n\cdot k)^\alpha$ for each emitted parton with momentum k. The $\mathcal{B}$ functions will contain poles in the analytic regulator α, which however will cancel in the product $\mathcal{B}_{q/q}\,\bar{\mathcal{B}}_{\bar{q}/\bar{q}}$. A remnant of this regulator dependence is the collinear anomaly [9], resulting in a dependence on the hard momentum transfer Q^2 in the product, which can be refactorized as:

$$\begin{aligned}&\left[\mathcal{B}_{q/q}(z_1,x_T^2,\mu)\,\bar{\mathcal{B}}_{\bar{q}/\bar{q}}(z_2,x_T^2,\mu)\right]_{Q^2} \quad (4)\\ &= \left(\frac{x_T^2Q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(L_\perp,\mu)} B_{q/q}(z_1,L_\perp,\mu)\,B_{\bar{q}/\bar{q}}(z_2,L_\perp,\mu)\,,\end{aligned}$$

where $L_\perp=\ln(x_T^2\mu^2/b_0^2)$. Note that $B_{q/q}=B_{\bar{q}/\bar{q}}$, we therefore do not distinguish them anymore. The $B_{i/j}$ functions can be regarded as the process-independent transverse PDFs, which can be matched onto the collinear ones for $x_T\ll 1/\Lambda_{\rm QCD}$ in the form

$$B_{i/j}(z,L_\perp,\mu) = \sum_k \int_z^1 \frac{d\xi}{\xi}\, I_{i/k}(\xi,L_\perp,\mu)\,\phi_{k/j}(z/\xi,\mu)\,. \quad (5)$$

We define the perturbative expansion of the $I_{i/j}$ functions as $I_{i/j}=\sum_n(\alpha_s/(4\pi))^n I_{i/j}^{(n)}$, and similarly for other functions. The second order coefficient $I_{q/q}^{(2)}$ is the main result in this Letter, while the other combinations of i and j will be presented in a forthcoming article.

At leading order (LO), it is clear that $\mathcal{B}_{q/q}^{(0)}(z,x_T^2,\mu)=\bar{\mathcal{B}}_{\bar{q}/\bar{q}}^{(0)}(z,x_T^2,\mu)=\delta(1-z)$. At NLO, these functions were calculated in [9] using a slightly different way of regularization. For our purpose, we need to recompute them with the regularization scheme of [15]. The results read

$$\begin{aligned}&\mathcal{B}_{q/q}^{(1)}(z,x_T^2,\mu) = C_F\, e^{(\epsilon+\alpha)L_\perp-(\epsilon+2\alpha)\gamma_E}\,\frac{\Gamma(-\epsilon-\alpha)}{\Gamma(1+\alpha)}\\ &\times\left(\frac{\bar{n}\cdot p_1}{\mu}\right)^\alpha\left(\frac{\nu}{\mu}\right)^\alpha(1-z)^{-1+\alpha}\,[4z+2(1-\epsilon)(1-z)^2],\\ &\bar{\mathcal{B}}_{\bar{q}/\bar{q}}^{(1)}(z,x_T^2,\mu) = C_F\, e^{\epsilon L_\perp-\epsilon\gamma_E}\,\Gamma(-\epsilon) \quad (6)\\ &\times\left(\frac{\mu}{n\cdot p_2}\right)^\alpha\left(\frac{\nu}{\mu}\right)^\alpha(1-z)^{-1-\alpha}\,[4z+2(1-\epsilon)(1-z)^2].\end{aligned}$$

It is easy to check that the poles in α vanish in the combination $\delta(1-z_1)\,\bar{\mathcal{B}}_{\bar{q}/\bar{q}}^{(1)}(z_2)+\mathcal{B}_{q/q}^{(1)}(z_1)\,\delta(1-z_2)$. The bare $F_{q\bar{q}}^{(1)}$ and $I_{q/q}^{(1)}$ functions can then be extracted from this combination following Eqs. (4) and (5). The remaining poles in the dimensional regulator $\epsilon=(4-d)/2$ can be renormalized in the $\overline{\rm MS}$ scheme:

$$\begin{aligned}&F_{i\bar{i}}^{\rm bare}(L_\perp) = Z_{i\bar{i}}^F(\mu)+F_{i\bar{i}}(L_\perp,\mu)\,, \quad (7)\\ &I_{i/j}^{\rm bare}(z,L_\perp) = \sum_k\int_z^1\frac{d\xi}{\xi}\,I_{i/k}(\xi,L_\perp,\mu)\,Z_{k/j}^I(z/\xi,L_\perp,\mu)\,.\end{aligned}$$

Note that for the renormalization at NNLO, we will need $I_{q/g}^{(1)}$ and $Z_{g/q}^{I,(1)}$, which we also computed.

At NNLO, the transverse PDFs receive 3 classes of contributions. In Figure 1, we show a typical Feynman diagram for each of them: (a) virtual+real diagrams; (b) double gluon emission diagrams; (c) quark-antiquark pair emission diagrams. The virtual+real diagrams are relatively easy to calculate. After carrying out the loop integrals, we encounter familiar integrals which already appeared at NLO. The virtual+real diagrams contain divergences requiring coupling constant renormalization, for which we include diagrams with the loop replaced by a counter-term. The main complication comes from the double real emission diagrams (b) and (c), where two propagators are raised to non-integer powers. We managed to reduce them to two-fold integrals involving hypergeometric functions. We then perform the remaining

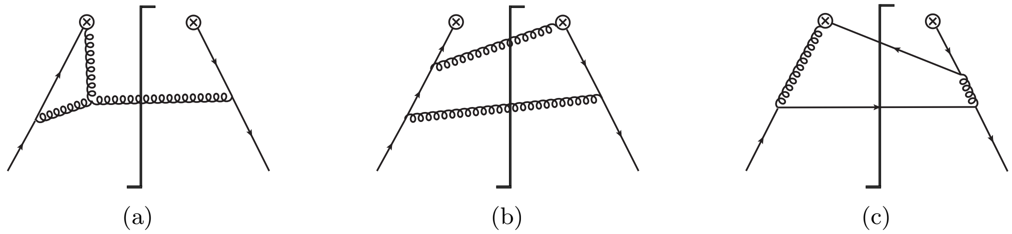

FIG. 1. Sample Feynman diagrams for our calculation: (a) virtual+real; (b) double gluon emission; (c) quark-antiquark pair emission.

integration by a systematic expansion in α and ϵ. To this end we made extensive use of properties of hypergeometric functions and the `HypExp` package [17]. The anti-collinear PDF $\bar{\mathcal{B}}_{\bar{q}/\bar{q}}$ can be obtained similarly, with $n \leftrightarrow \bar{n}$. Note however that the analytic regulator is still given by $(\nu/n\cdot k)^\alpha$.

The $\mathcal{B}^{(2)}_{q/q}$ and $\bar{\mathcal{B}}^{(2)}_{\bar{q}/\bar{q}}$ functions contain poles in the analytic regulator α. It is important to verify that these poles cancel in the combination $\delta(1-z_1)\,\bar{\mathcal{B}}^{(2)}_{\bar{q}/\bar{q}}(z_2) + \mathcal{B}^{(2)}_{q/q}(z_1)\,\delta(1-z_2) + \mathcal{B}^{(1)}_{q/q}(z_1)\,\bar{\mathcal{B}}^{(1)}_{\bar{q}/\bar{q}}(z_2)$. We therefore show below the singular structures in these two functions:

$$
\begin{aligned}
\mathcal{B}^{(2)}_{q/q}(z,L_\perp) &= 4C_F\, e^{2(\epsilon+\alpha)L_\perp} \left(\frac{\bar{n}\cdot p_1}{\mu}\right)^{2\alpha} \left(\frac{\nu}{\mu}\right)^{2\alpha} \\
&\times \Bigg\{ \frac{P^{(0)}_{qq}(z)}{\alpha}\left(\frac{1}{\epsilon^2}+\zeta_2\right) - \frac{2C_F(1-z)}{\epsilon\,\alpha} + \frac{\delta(1-z)}{\alpha} \\
&\quad\times \Bigg[C_F\left[\left(\frac{2}{\alpha}-3\right)\left(\frac{1}{\epsilon^2}+\zeta_2\right) - \frac{4}{\epsilon^3} + \frac{4\zeta_3}{3}\right] + C_A\bigg(\frac{1}{2\epsilon^3} \\
&\quad - \frac{67}{36\epsilon} - \frac{101}{27} + \frac{23\zeta_3}{6}\bigg) + T_f n_f\left(\frac{5}{9\epsilon}+\frac{28}{27}\right) \\
&\quad - \frac{\beta_0}{4}\left(\frac{1}{\epsilon^2}+\zeta_2\right)\Bigg]\Bigg\} + \mathcal{B}^{(2),\mathrm{ct}}_{q/q}(z,L_\perp) + \mathcal{O}(\alpha^0)\,,
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\bar{\mathcal{B}}^{(2)}_{\bar{q}/\bar{q}}(z,L_\perp) &= 4C_F\, e^{2\epsilon L_\perp} \left(\frac{\mu}{n\cdot p_2}\right)^{2\alpha} \left(\frac{\nu}{\mu}\right)^{2\alpha} \\
&\times \Bigg\{ -\frac{P^{(0)}_{qq}(z)}{\alpha}\left(\frac{1}{\epsilon^2}+\zeta_2\right) + \frac{2C_F(1-z)}{\epsilon\,\alpha} + \frac{\delta(1-z)}{\alpha} \\
&\quad\times \Bigg[C_F\left(\frac{2}{\alpha}+3\right)\left(\frac{1}{\epsilon^2}+\zeta_2\right) - C_A\bigg(\frac{1}{2\epsilon^3} - \frac{67}{36\epsilon} - \frac{101}{27} \\
&\quad + \frac{23\zeta_3}{6}\bigg) - T_f n_f\left(\frac{5}{9\epsilon}+\frac{28}{27}\right) + \frac{\beta_0}{4}\left(\frac{1}{\epsilon^2}+\zeta_2\right)\Bigg]\Bigg\} \\
&- \mathcal{B}^{(2),\mathrm{ct}}_{q/q}(z,L_\perp) + \mathcal{O}(\alpha^0)\,,
\end{aligned}
\tag{9}
$$

where $P^{(0)}_{qq}(z) = 2C_F[(1+z^2)/(1-z)]_+$, $\beta_0 = 11C_A/3 - 4T_F n_f/3$. The function $\mathcal{B}^{(2),\mathrm{ct}}_{q/q}$ comes from α_s renormalization and is given by

$$
\mathcal{B}^{(2),\mathrm{ct}}_{q/q}(z,L_\perp) = 4C_F\, e^{\epsilon L_\perp}\, \frac{\delta(1-z)}{\alpha}\, \beta_0 \left(\frac{1}{\epsilon^2}+\frac{\zeta_2}{2}\right). \tag{10}
$$

From the above formulae, it is clear that the pole terms with the color structures $C_F C_A$ and $C_F T_F n_f$ vanish in the sum $\delta(1-z_1)\,\bar{\mathcal{B}}^{(2)}_{\bar{q}/\bar{q}}(z_2) + \mathcal{B}^{(2)}_{q/q}(z_1)\,\delta(1-z_2)$. The remaining singularities are canceled by the product $\mathcal{B}^{(1)}_{q/q}(z_1)\,\bar{\mathcal{B}}^{(1)}_{\bar{q}/\bar{q}}(z_2)$.

We are now ready to extract the functions $F^{(2)}_{q\bar{q}}$ and $I^{(2)}_{q/q}$, following the procedure of Eqs. (4) and (5) and carrying out the renormalization as in Eq. (7). We have checked that the $F^{(2)}_{q\bar{q}}$ function extracted from our calculation agrees with the expression given in [9] and that the $I^{(2)}_{q/q}$ function satisfies the RG equation

$$
\begin{aligned}
\frac{dI_{q/q}(z,L_\perp,\mu)}{d\ln\mu} &= \Big[\Gamma^F_{\mathrm{cusp}}(\alpha_s)L_\perp - 2\gamma^q(\alpha_s)\Big] I_{q/q}(z,L_\perp,\mu) \\
&\quad - \sum_k 2\int_z^1 \frac{d\xi}{\xi}\, I_{q/k}(\xi,L_\perp,\mu)\, P_{kq}(z/\xi,\mu)\,,
\end{aligned}
\tag{11}
$$

where the anomalous dimensions Γ^F_{cusp} and γ^q up to 2 loops can be found in [9], and the splitting functions P_{ij} up to 2 loop order can be found in [18]. The finiteness of $I^{(2)}_{q/q}$ and its RG properties demonstrate, for the first time, that the operator definition for the transverse PDFs supplemented with the analytic regulator is valid beyond the first non-trivial order.

We finally give the scale-independent part of the $I^{(2)}_{q/q}$ function, which is the main result of this Letter. It can be written as

$$
\begin{aligned}
I^{(2)}_{q/q}(z,0) &= \delta(1-z)\bigg[C_F^2\frac{5\zeta_4}{4} + C_F C_A\bigg(\frac{3032}{81} - \frac{67\zeta_2}{6} \\
&\quad - \frac{266\zeta_3}{9} + 5\zeta_4\bigg) + C_F T_F n_f\bigg(-\frac{832}{81} + \frac{10\zeta_2}{3} + \frac{28\zeta_3}{9}\bigg)\bigg] \\
&\quad + P^{(0)}_{qq}(z)\bigg[C_F\bigg(12\zeta_3 + 4H_0 + \frac{3}{2}H_{0,0} + 4H_{0,1,0} + 2H_{0,1,1} \\
&\quad - 2H_{1,0,0} + 4H_{1,0,1} + 4H_{1,1,0}\bigg) + C_A\bigg(\zeta_3 - \frac{202}{27} - \frac{38}{9}H_0 \\
&\quad - \frac{11}{6}H_{0,0} - H_{0,0,0} - 2H_{0,1,0} - 2H_{1,0,1} - 2H_{1,1,0}\bigg) \\
&\quad + T_F n_f\bigg(\frac{56}{27} + \frac{10}{9}H_0 + \frac{2}{3}H_{0,0}\bigg)\bigg]
\end{aligned}
$$

$$+ P_{qg}^{(0)}(z) C_F \left[-\frac{68}{27} + \frac{4\zeta_2}{3} + \frac{32}{9} H_0 - \frac{4}{3} H_{0,0} + \frac{4}{3} H_{1,0} \right]$$
$$+ P_{gq}^{(0)}(z) T_F \left[\frac{86}{27} - \frac{4\zeta_2}{3} - \frac{4}{3} H_{1,0} \right]$$
$$+ C_F^2 \bigg[(2-24z) H_0 + (3+7z) H_{0,0} + 2(1+z) H_{0,0,0}$$
$$+ 2z H_1 + (1-z)\Big(6\zeta_2 - 22 + 4H_{0,1} + 12 H_{1,0} \Big) \bigg]$$
$$+ C_F C_A \bigg[(2+10z) H_0 - 4z H_{0,0} - 2z H_1$$
$$+ (1-z) \left(\frac{44}{3} - 6\zeta_2 - 4 H_{1,0} \right) \bigg]$$
$$+ C_F T_F \bigg[\frac{-50+38z}{9} + \frac{20+8z}{9} H_0 + \frac{2-22z}{3} H_{0,0}$$
$$+ 4(1+z) H_{0,0,0} \bigg] - \frac{4}{3} C_F T_F n_f (1-z) \,, \qquad (12)$$

where $P_{qg}^{(0)}(z) = 2T_F[z^2 + (1-z)^2]$, $P_{gq}^{(0)}(z) = 2C_F[1 + (1-z)^2]/z$, and $H_{\{m\}} \equiv H(\{m\}, z)$ are harmonic polylogarithms introduced in [19].

The coefficient function C_{qq} relevant for the Drell-Yan process is related to $I_{q/q}$ by [9]

$$C_{qq}(z, \alpha_s(\mu_b)) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/q}(z, 0, \alpha_s(\mu_b)) \,, \quad (13)$$

where the function C_V is determined from the virtual corrections to the Drell-Yan process and can be extracted from [20]. The NNLO expression for C_{qq} reads

$$C_{qq}^{(2)}(z, \alpha_s(\mu_b)) = I_{q/q}^{(2)}(z, 0) + C_F^2\,(1-z)\,(14\zeta_2 - 16)$$
$$+ C_F\, \delta(1-z) \times \left[C_F \left(\frac{255}{8} - 19\zeta_2 - 30\zeta_3 + \frac{87\zeta_4}{4} \right) \right.$$
$$+ C_A \left(-\frac{51157}{648} + \frac{1061\zeta_2}{18} + \frac{313\zeta_3}{9} - 8\zeta_4 \right)$$
$$\left. + T_F n_f \left(\frac{4085}{162} - \frac{182\zeta_2}{9} + \frac{4\zeta_3}{9} \right) \right] . \qquad (14)$$

Starting from this expression, we have checked that we can reproduce the $\mathcal{H}_{q\bar{q}\leftarrow q\bar{q}}^{(2)}$ function in [8]. Note that $I_{q/q}$ is universal, while C_{qq} and $\mathcal{H}_{q\bar{q}\leftarrow q\bar{q}}$ contain both process-independent and process-dependent parts [7]. It is straightforward to compute the C_{qq} and $\mathcal{H}_{q\bar{q}\leftarrow q\bar{q}}$ functions from our results up to NNLO for any $q\bar{q}$ initiated process given the knowledge of the two-loop virtual corrections.

In conclusion, we have calculated the perturbative quark-to-quark transverse PDF at NNLO based on a gauge invariant operator definition with an analytic regulator. We demonstrate for the first time that such a definition works beyond the first non-trivial order. We extract from our calculation the coefficient functions relevant for a N^{3}LL Q_T resummation. Our results can be applied to all quark-antiquark annihilation processes yielding a colorless final state, provided the NNLO virtual corrections are known. Combined with the recent work [21], our results could also be applied to the $q\bar{q} \to t\bar{t}$ process. Our method of calculation can be easily extended to all parton combinations, which will be presented in a forthcoming article.

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